S382
Equations booklet

Contents

This complete list of constants, conversion factors and equations is included for reference. It may be useful as an aid to your memory but please remember that many of the entries will *not* be needed in the examination.

An identical list of constants, conversion factors and equations will be attached to the examination.

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2.1 PT/TEX

1 Constants and conversions

Table 1 Common SI unit conversions and derived units.

Quantity	Unit	Conversion
speed	$\mathrm{m\,s^{-1}}$	
acceleration	$\mathrm{m\,s^{-2}}$	
angular speed	$\mathrm{rad\,s^{-1}}$	
angular acceleration	$\mathrm{rad\,s^{-2}}$	
linear momentum	$\mathrm{kg\,m\,s^{-1}}$	
angular momentum	$\mathrm{kg\,m^2\,s^{-1}}$	
force	newton (N)	$1\,\mathrm{N} = 1\,\mathrm{kg\,m\,s^{-2}}$
energy	joule (J)	$1\,\mathrm{J} = 1\,\mathrm{N\,m} = 1\,\mathrm{kg\,m^2\,s^{-2}}$
power	watt (W)	$1\,\mathrm{W} = 1\,\mathrm{J\,s^{-1}} = 1\,\mathrm{kg\,m^2\,s^{-3}}$
pressure	pascal (Pa)	$1\,\mathrm{Pa} = 1\,\mathrm{N\,m^{-2}} = 1\,\mathrm{kg\,m^{-1}\,s^{-2}}$
frequency	hertz (Hz)	$1\,\mathrm{Hz} = 1\,\mathrm{s^{-1}}$
charge	coulomb (C)	$1\,\mathrm{C} = 1\,\mathrm{A\,s}$
potential difference	volt (V)	$1\,\mathrm{V} = 1\,\mathrm{J\,C^{-1}} = 1\,\mathrm{kg\,m^2\,s^{-3}\,A^{-1}}$
electric field	$\mathrm{N\,C^{-1}}$	$1\,\mathrm{N\,C^{-1}} = 1\,\mathrm{V\,m^{-1}} = 1\,\mathrm{kg\,m\,s^{-3}\,A^{-1}}$
magnetic field	tesla (T)	$1\,\mathrm{T} = 1\,\mathrm{N\,s\,m^{-1}\,C^{-1}} = 1\,\mathrm{kg\,s^{-2}\,A^{-1}}$

Table 2 Other unit conversions.

wavelength
1 nanometre (nm) $= 10\,\text{Å} = 10^{-9}\,\mathrm{m}$
1 ångstrom $= 0.1\,\mathrm{nm} = 10^{-10}\,\mathrm{m}$

angular measure
$1° = 60\,\mathrm{arcmin} = 3600\,\mathrm{arcsec}$
$1° = 0.017\,45$ radian
1 radian $= 57.30°$

temperature
absolute zero: $0\,\mathrm{K} = -273.15\,°\mathrm{C}$
$0\,°\mathrm{C} = 273.15\,\mathrm{K}$

spectral flux density
1 jansky (Jy) $= 10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$
$1\,\mathrm{W\,m^{-2}\,Hz^{-1}} = 10^{26}\,\mathrm{Jy}$

cgs units
$1\,\mathrm{erg} = 10^{-7}\,\mathrm{J}$
$1\,\mathrm{dyne} = 10^{-5}\,\mathrm{N}$
$1\,\mathrm{gauss} = 10^{-4}\,\mathrm{T}$
$1\,\mathrm{emu} = 10\,\mathrm{C}$

mass–energy equivalence
$1\,\mathrm{kg} = 8.99 \times 10^{16}\,\mathrm{J}/c^2$ (c in $\mathrm{m\,s^{-1}}$)
$1\,\mathrm{kg} = 5.61 \times 10^{35}\,\mathrm{eV}/c^2$ (c in $\mathrm{m\,s^{-1}}$)

distance
1 astronomical unit (AU) $= 1.496 \times 10^{11}\,\mathrm{m}$
1 light-year (ly) $= 9.461 \times 10^{15}\,\mathrm{m} = 0.307\,\mathrm{pc}$
1 parsec (pc) $= 3.086 \times 10^{16}\,\mathrm{m} = 3.26\,\mathrm{ly}$

energy
$1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$
$1\,\mathrm{J} = 6.242 \times 10^{18}\,\mathrm{eV}$

cross-sectional area
$1\,\mathrm{barn} = 10^{-28}\,\mathrm{m^2}$
$1\,\mathrm{m^2} = 10^{28}\,\mathrm{barn}$

pressure
$1\,\mathrm{bar} = 10^5\,\mathrm{Pa}$
$1\,\mathrm{Pa} = 10^{-5}\,\mathrm{bar}$
1 atmosphere $= 1.013\,25$ bar
1 atmosphere $= 1.013\,25 \times 10^5\,\mathrm{Pa}$

Table 3 Constants.

Name of constant	Symbol	SI value
Fundamental constants		
gravitational constant	G	$6.673 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Boltzmann's constant	k	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
fine structure constant	$\alpha = e^2/4\pi\varepsilon_0\hbar c$	$1/137.0$
Stefan–Boltzmann constant	σ	$5.671 \times 10^{-8}\,\mathrm{J\,m^{-2}\,K^{-4}\,s^{-1}}$
Thomson cross-section	σ_{T}	$6.652 \times 10^{-29}\,\mathrm{m^2}$
permittivity of free space	ε_0	$8.854 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$
Particle constants		
charge of proton	e	$1.602 \times 10^{-19}\,\mathrm{C}$
charge of electron	$-e$	$-1.602 \times 10^{-19}\,\mathrm{C}$
electron rest mass	m_{e}	$9.109 \times 10^{-31}\,\mathrm{kg}$ $= 0.511\,\mathrm{MeV}/c^2$
proton rest mass	m_{p}	$1.673 \times 10^{-27}\,\mathrm{kg}$ $= 938.3\,\mathrm{MeV}/c^2$
neutron rest mass	m_{n}	$1.675 \times 10^{-27}\,\mathrm{kg}$ $= 939.6\,\mathrm{MeV}/c^2$
atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg}$
Astronomical constants		
mass of the Sun	$\mathrm{M}_\odot$	$1.99 \times 10^{30}\,\mathrm{kg}$
radius of the Sun	$\mathrm{R}_\odot$	$6.96 \times 10^{8}\,\mathrm{m}$
luminosity of the sun	$\mathrm{L}_\odot$	$3.83 \times 10^{26}\,\mathrm{W}$
mass of the Earth	$\mathrm{M}_\oplus$	$5.97 \times 10^{24}\,\mathrm{kg}$
radius of the Earth	$\mathrm{R}_\oplus$	$6.37 \times 10^{6}\,\mathrm{m}$
mass of Jupiter	$\mathrm{M_J}$	$1.90 \times 10^{27}\,\mathrm{kg}$
radius of Jupiter	$\mathrm{R_J}$	$7.15 \times 10^{7}\,\mathrm{m}$
astronomical unit	AU	$1.496 \times 10^{11}\,\mathrm{m}$
light-year	ly	$9.461 \times 10^{15}\,\mathrm{m}$
parsec	pc	$3.086 \times 10^{16}\,\mathrm{m}$
Hubble parameter	H_0	$(70.4 \pm 1.5)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ $(2.28 \pm 0.05) \times 10^{-18}\,\mathrm{s^{-1}}$
age of Universe	t_0	$(13.73 \pm 0.15) \times 10^{9}$ years
current critical density	$\rho_{\mathrm{c},0}$	$(9.30 \pm 0.40) \times 10^{-27}\,\mathrm{kg\,m^{-3}}$
current dark energy density	$\Omega_{\Lambda,0}$	$(73.2 \pm 1.8)\%$
current matter density	$\Omega_{\mathrm{m},0}$	$(26.8 \pm 1.8)\%$
current baryonic matter density	$\Omega_{\mathrm{b},0}$	$(4.4 \pm 0.2)\%$
current non-baryonic matter density	$\Omega_{\mathrm{c},0}$	$(22.3 \pm 0.9)\%$
current curvature density	$\Omega_{\mathrm{k},0}$	$(-1.4 \pm 1.7)\%$
current deceleration	q_0	-0.595 ± 0.025

2 Mathematics

2.1 Algebra

$y^a \times y^b = y^{a+b}, \quad y^a/y^b = y^{a-b}, \quad (y^a)^b = y^{ab}$

For logarithms to any base:

$\log(a \times b) = \log a + \log b, \quad \log(a/b) = \log a - \log b, \quad \log a^b = b \log a$

$\sin^2\theta + \cos^2\theta = 1$

$\boldsymbol{a} \cdot \boldsymbol{b} = ab\cos\theta = a_x b_x + a_y b_y + a_z b_z$ scalar product
$|\boldsymbol{a} \times \boldsymbol{b}| = ab\sin\theta$ vector product magnitude
$\boldsymbol{a} \times \boldsymbol{b} = (a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)$ vector product components

2.2 Calculus

if $y = at^n$ then $\mathrm{d}y/\mathrm{d}t = nat^{n-1}$
if $y = a\exp(kt)$ then $\mathrm{d}y/\mathrm{d}t = ak\exp(kt)$
if $y = a\sin(\omega t)$ then $\mathrm{d}y/\mathrm{d}t = a\omega\cos(\omega t)$
if $y = a\cos(\omega t)$ then $\mathrm{d}y/\mathrm{d}t = -a\omega\sin(\omega t)$
if $y = a\log_\mathrm{e} t$ then $\mathrm{d}y/\mathrm{d}t = a/t$

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u} \times \dfrac{\mathrm{d}u}{\mathrm{d}x}$ chain rule

$\dfrac{\mathrm{d}(u+v)}{\mathrm{d}t} = \dfrac{\mathrm{d}u}{\mathrm{d}t} + \dfrac{\mathrm{d}v}{\mathrm{d}t}$ sum rule

$\dfrac{\mathrm{d}(uv)}{\mathrm{d}t} = u\dfrac{\mathrm{d}v}{\mathrm{d}t} + v\dfrac{\mathrm{d}u}{\mathrm{d}t}$ product rule

$\dfrac{\mathrm{d}(u/v)}{\mathrm{d}t} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}t} - u\frac{\mathrm{d}v}{\mathrm{d}t}}{v^2}$ quotient rule

$\dfrac{\mathrm{d}(\log_\mathrm{e} x)}{\mathrm{d}t} = \dfrac{\dot{x}}{x}$ logarithmic differentiation

$f(x) \approx f(a) + (x-a)f'(a) + (x-a)^2 f''(a)/2! + \ldots$ Taylor series

$\mathrm{grad}\ T = \nabla T = \left(\dfrac{\partial T}{\partial x}, \dfrac{\partial T}{\partial y}, \dfrac{\partial T}{\partial z}\right)$ gradient of a scalar field

$\mathrm{div}\ \boldsymbol{A} = \nabla \cdot \boldsymbol{A} = \dfrac{\partial A_x}{\partial x} + \dfrac{\partial A_y}{\partial y} + \dfrac{\partial A_z}{\partial z}$ divergence of a vector field

if $y = at^n$ then $\int y\,\mathrm{d}t = \dfrac{at^{n+1}}{n+1} + C$ (for $n \neq -1$)
if $y = at^{-1}$ then $\int y\,\mathrm{d}t = a\log_\mathrm{e} t + C$
if $y = a\exp(kt)$ then $\int y\,\mathrm{d}t = \dfrac{a\exp(kt)}{k} + C$
if $y = a\sin(\omega t)$ then $\int y\,\mathrm{d}t = -\dfrac{a\cos(\omega t)}{\omega} + C$
if $y = a\cos(\omega t)$ then $\int y\,\mathrm{d}t = \dfrac{a\sin(\omega t)}{\omega} + C$

$\int (u+v)\,\mathrm{d}t = \int u\,\mathrm{d}t + \int v\,\mathrm{d}t$ sum rule for integration

$\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u$ integration by parts

2.3 Geometry

straight-line graph	area of a circle	volume of a sphere	surface area of a sphere
$y = mx + c$	$A = \pi R^2$	$V = 4\pi R^3/3$	$A = 4\pi R^2$

2.4 Statistics

$\sigma_N = \sqrt{N}$ Poisson statistics

3 Physics

3.1 Mechanics

$m_{\mathrm{r}} = m_{\mathrm{A}} m_{\mathrm{B}}/(m_{\mathrm{A}} + m_{\mathrm{B}})$	reduced mass
$\boldsymbol{v} = \mathrm{d}\boldsymbol{r}/\mathrm{d}t$	velocity
$\boldsymbol{p} = m\boldsymbol{v}$	linear momentum
$\boldsymbol{a} = \mathrm{d}\boldsymbol{v}/\mathrm{d}t$	acceleration
$\boldsymbol{F} = m\boldsymbol{a}$	Newton's second law
$F_{\mathrm{c}} = mv^2/r = mr\omega^2$	magnitude of centrifugal/centripetal force
$a = GM/R^2$	magnitude of gravitational acceleration
$v_{\mathrm{esc}} = (2GM/R)^{1/2}$	escape speed
$E_{\mathrm{K}} = mv^2/2 = p^2/2m$	kinetic energy
$E_{\mathrm{GR}} = -GmM/r$	gravitational potential energy
$E_{\mathrm{GR}} = -3GM^2/5R$	gravitational potential energy of uniform sphere
$E_{\mathrm{rot}} = I\omega^2/2$	rotational energy of a sphere
$I = 2MR^2/5$	moment of inertia of a uniform sphere
$\boldsymbol{j} = \boldsymbol{R} \times \boldsymbol{p}$	angular momentum (particle)
$\boldsymbol{J} = I\boldsymbol{\omega}$	angular momentum (body)
$\boldsymbol{F}_{21} = \dfrac{-Gm_1m_2}{r^2}\widehat{\boldsymbol{r}}$	Newton's law of gravitation

3.2 Special relativity

$E_0 = mc^2$	mass energy
$E_{\mathrm{K}} = (\gamma - 1)mc^2$	kinetic energy
$\boldsymbol{p} = \gamma m\boldsymbol{v}$	linear momentum
$E = \gamma mc^2 = E_0 + E_{\mathrm{K}}$	total energy
$E^2 = p^2c^2 + m^2c^4$	energy–momentum relation
$x' = \gamma(V)(x - Vt)$	Lorentz transformation (space)
$t' = \gamma(V)(t - Vx/c^2)$	Lorentz transformation (time)
$\gamma(V) = \dfrac{1}{\sqrt{1 - (V^2/c^2)}}$	Lorentz factor

3.3 Gases

$PV = NkT$	ideal gas law	$P = \rho kT/\overline{m} = \rho kT/\mu u$
$PV^{\gamma} = \text{constant}$	adiabatic process	$P \propto \rho^{\gamma}$
$c_{\mathrm{s}} \sim (P/\rho)^{1/2}$	isothermal sound speed	$c_{\mathrm{s}} \sim 10\ \left(T/10^4\ \mathrm{K}\right)^{1/2}\ \mathrm{km\ s^{-1}}$

pressure due to gas:

$$P_{\mathrm{NR}} = \frac{2}{3}\frac{E_{\mathrm{K}}}{V} \quad \text{non-relativistic} \qquad P_{\mathrm{UR}} = \frac{1}{3}\frac{E_{\mathrm{K}}}{V} \quad \text{ultra-relativistic}$$

convection condition:

$$\frac{\mathrm{d}\log_{\mathrm{e}} T}{\mathrm{d}\log_{\mathrm{e}} P} > \frac{(\gamma - 1)}{\gamma}$$

adiabatic index:

$$\gamma = \frac{1 + (s/2)}{(s/2)}$$

$\rho = N\overline{m}/V = n\overline{m} = n\mu u$	density
$P_{\mathrm{rad}} = 4\sigma T^4/3c$	radiation pressure
$\kappa(r) = \kappa_0 \rho(r) T^{-3.5}(r)$	Kramer's opacity
$\mu = \sum_i n_i \frac{m_i}{u} / \sum_i n_i$	mean molecular mass
$n_i = \rho X_i/m_i$	number density of nucleons
$n_{\mathrm{e}} = \rho X_{\mathrm{e}}/m_{\mathrm{e}} = \rho Y_{\mathrm{e}}/m_{\mathrm{H}}$	number density of electrons

3.4 Radiation

$B_\nu(T) = \dfrac{2h\nu^3}{c^2}\dfrac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}$	black body radiation	$B_\lambda(T) = \dfrac{2hc^2}{\lambda^5}\dfrac{1}{\exp\left(\frac{hc}{\lambda kT}\right)-1}$
$B_\nu(T) = \dfrac{2kT\nu^2}{c^2}$	Rayleigh–Jeans law	$B_\lambda(T) = \dfrac{2ckT}{\lambda^4}$
$B_\nu(T) = \dfrac{2h\nu^3}{c^2}\exp\left(-\dfrac{h\nu}{kT}\right)$	Wien displacement law	$B_\lambda(T) = \dfrac{2hc^2}{\lambda^5}\exp\left(-\dfrac{hc}{\lambda kT}\right)$

$\lambda_{\max}T = 5.1 \times 10^{-3}$ m K	Wien displacement law (maximizing B_ν)
$\lambda_{\max}T = 2.9 \times 10^{-3}$ m K	Wien displacement law (maximizing B_λ)
$F = \sigma T^4$	Stefan–Boltzmann law
$\langle E_{\rm ph}\rangle = 2.70kT$	mean photon energy of black body
$c = \lambda\nu$	speed of light
$\Delta\lambda/\lambda = V/c$	Doppler shift of light

3.5 Quantum physics

$E_{\rm ph} = \Delta E_{\rm atom} = h\nu = hc/\lambda = cp_{\rm ph}$	photon energy
$\lambda_{\rm dB} = h/p = h/(3mkT)^{1/2}$	de Broglie wavelength
$E_n = -13.60\ {\rm eV}/n^2$	hydrogen energy levels
$\mu' = mc^2 - kT\log_{\rm e}(g_{\rm s}n_{\rm QNR}/n)$	chemical potential
$n_{\rm QNR} = (2\pi mkT/h^2)^{3/2}$	quantum concentration (non-relativistic)
$P_{\rm NR} = K_{\rm NR}n_{\rm e}^{5/3}$	degenerate electron gas pressure (non-relativistic) where $K_{\rm NR} = (h^2/5m_{\rm e})(3/8\pi)^{2/3}$
$P_{\rm UR} = K_{\rm UR}n_{\rm e}^{4/3}$	degenerate electron gas pressure (ultra-relativistic) where $K_{\rm UR} = (hc/4)(3/8\pi)^{1/3}$
$\left(-\dfrac{\hbar^2}{2m_{\rm r}}\dfrac{\partial^2}{\partial r^2} + V(r)\right)\psi_{\rm s}(r) = E\psi_{\rm s}(r)$	time-independent one-dimensional Schrödinger equation

Fermi energy: $E_{\rm F} \approx p_{\rm F}^2/2m$, Fermi momentum: $p_{\rm F} = (3n_{\rm e}/8\pi)^{1/3}h$,
Fermi temperature: $T_{\rm F} = n^{2/3}h^2/(2\pi mk)$

3.6 Nuclear fusion

$P_{\rm pen} \approx \exp[-(E_{\rm G}/E)^{1/2}]$	barrier penetration probability
$E_{\rm G} = 2m_{\rm r}c^2(\pi\alpha Z_{\rm A}Z_{\rm B})^2$	Gamow energy
$l = 1/\sigma n$	mean free path
$\sigma(E) = \dfrac{S(E)}{E}\exp\left[-\left(\dfrac{E_{\rm G}}{E}\right)^{1/2}\right]$	reaction cross-section
$R_{\rm AB} = n_{\rm A}n_{\rm B}\langle\sigma v_{\rm r}\rangle$	fusion rate (dissimilar particles)
$R_{\rm AA} = \left(n_{\rm A}^2/2\right)\langle\sigma v_{\rm r}\rangle$	fusion rate (similar particles)
$\tau_{\rm A} = n_{\rm A}/R_{\rm AB}$	mean lifetime (dissimilar particles)
$\tau_{\rm A} = n_{\rm A}/2R_{\rm AA}$	mean lifetime (similar particles)
$E_0 = \left(E_{\rm G}(kT/2)^2\right)^{1/3}$	energy of Gamow peak
$\Delta \sim 1.8(E_{\rm G}/kT)^{1/6}kT$	Gamow width
$R_{\rm AB} \propto T^\nu$	integrated fusion rate
$\nu = \left(\dfrac{E_{\rm G}}{4kT}\right)^{1/3} - \dfrac{2}{3}$	temperature exponent of fusion rate
$\varepsilon(r) = \Delta m\,c^2 \times R_{\rm AB} = \varepsilon_0\rho^2(r)T^\nu(r)$	energy generation rate

$$R_{\rm AB} = \frac{6.48\times10^{-24}}{A_{\rm r}Z_{\rm A}Z_{\rm B}} \times \frac{n_{\rm A}n_{\rm B}}{[{\rm m}^{-6}]} \times \frac{S(E_0)}{[\text{keV barns}]} \times \left(\frac{E_{\rm G}}{4kT}\right)^{2/3} \times \exp\left[-3\left(\frac{E_{\rm G}}{4kT}\right)^{1/3}\right]\ {\rm m}^{-3}\ {\rm s}^{-1}$$

integrated fusion rate per unit volume

4 Astrophysics

4.1 Basic astronomy

$F = L/4\pi d^2$	flux
$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$	luminosity
$m_1 - m_2 = 2.5 \log_{10}(F_2/F_1)$	apparent magnitude
$M = m + 5 - 5 \log_{10} d - A$	absolute and apparent magnitude
$M_1 - M_2 = 2.5 \log_{10}(L_2/L_1)$	absolute magnitude
$d_{\max} = (L/4\pi S)^{1/2}$	maximum distance / limiting flux relationship
$N(S) = (4\pi n_0/3)\,(L/4\pi S)^{3/2}$	number of detectable sources
$I(\mu)/I(1) = 1 - u(1-\mu)$	linear limb darkening law

4.2 Self-gravitating systems and stellar structure

$\tau_{\mathrm{ff}} = (3\pi/32G\rho)^{1/2}$	free-fall time
$\tau_{\mathrm{KH}} = GM^2/RL$	Kelvin–Helmholtz timescale
$\tau_{\mathrm{nuc}} = E_{\mathrm{fusion}}/L$	nuclear lifetime
$\mathrm{d}P(r)/\mathrm{d}r = -Gm(r)\rho(r)/r^2$	hydrostatic equilibrium
$\mathrm{d}m(r)/\mathrm{d}r = 4\pi r^2 \rho(r)$	mass continuity
$\mathrm{d}T/\mathrm{d}r = -[3\kappa(r)\rho(r)L(r)]/[(4\pi r^2)(16\sigma)T^3(r)]$	radiative diffusion
$\mathrm{d}L/\mathrm{d}r = 4\pi r^2 \varepsilon(r)$	energy generation
$\langle P \rangle = -\frac{1}{3} E_{\mathrm{GR}}/V$	virial theorem (general)
$2E_{\mathrm{K}} + E_{\mathrm{GR}} = 0$	virial theorem (non-relativistic)
$E_{\mathrm{K}} + E_{\mathrm{GR}} = 0$	virial theorem (ultra-relativistic)
$E_{\mathrm{TOT}} = E_{\mathrm{K}} + E_{\mathrm{GR}}$	total energy
$P_{\mathrm{c}} \sim (\pi/36)^{1/3} G M^{2/3} \rho_{\mathrm{c}}^{4/3}$	Clayton stellar model
Jeans mass: $M_{\mathrm{J}} = 3kTR_{\mathrm{J}}/2G\overline{m}$	(NB this is *not* the mass of Jupiter $\mathrm{M_J}$.)

Jeans density:

$$\rho_{\mathrm{J}} = \left(\frac{3}{4\pi M_{\mathrm{J}}^2}\right)\left(\frac{3kT}{2G\overline{m}}\right)^3$$

(NB this is *not* the density of Jupiter, for which ρ_{J} is also used.)

4.3 Pulsars

$\dot{E}_{\mathrm{rad}} = (2/3c^3)(\mu_0/4\pi)m^2\omega^4 \sin^2\theta$	magnetic dipole radiation
$\dot{E}_{\mathrm{rot}} = I\omega\dot{\omega}$	rotational energy loss
$B = \mu_0 m/4\pi R^3$	surface magnetic field
$B/\mathrm{tesla} \geq 3.3 \times 10^{15}(P\dot{P}/\mathrm{seconds})^{1/2}$	magnetic-field–period relation
$t < -\frac{1}{2}(\omega/\dot{\omega}) \equiv \frac{1}{2}\left(P/\dot{P}\right)$	age–period relation

4.4 Orbits

$\omega = 2\pi/P$	orbital angular speed
$\phi = [(t - T_0)/P] - N_{\mathrm{orb}}$	orbital phase
$a^3/P^2 = G(M_* + M_{\mathrm{P}})/4\pi^2$	Kepler's third law
$M_* \boldsymbol{r}_* = -M_{\mathrm{P}} \boldsymbol{r}_{\mathrm{P}}$	reflex orbit & planetary orbit
$A_{\mathrm{RV}} = 2\pi a M_{\mathrm{P}} \sin i/(M_* + M_{\mathrm{P}})P(1-e^2)^{1/2}$	radial velocity amplitude
$M_{\mathrm{P}} \sin i = A_{\mathrm{RV}}\left(M_*^2 P/2\pi G\right)^{1/3}$	mass–radial-velocity relation
$J = M_1 M_2 \left[Ga/(M_1 + M_2)\right]^{1/2}$	orbital angular momentum
$\tau_{\mathrm{circ}} = (2/21)(Q_{\mathrm{P}}/k_{\mathrm{dP}})\left(a^3/GM_*\right)^{1/2}(M_{\mathrm{P}}/M_*)\,(a/R_{\mathrm{P}})^5$	circularization timescale
$R_{\mathrm{H}} = a(M_{\mathrm{P}}/3M_*)^{1/3}$	radius of Hill sphere
$d_{\mathrm{R}} = R_{\mathrm{M}}(2M_{\mathrm{P}}/M_{\mathrm{M}})^{1/3}$	Roche limit

$$V = V_0 + \frac{2\pi a M_{\mathrm{P}} \sin i}{(M_{\mathrm{P}} + M_*)P(1-e^2)^{1/2}}\left[\cos(\theta + \omega_{\mathrm{OP}}) + e\cos\omega_{\mathrm{OP}}\right]$$

stellar radial velocity

4.5 Exoplanets

$\Delta F/F = R_\mathrm{P}^2/R_*^2$	planetary transit light reduction
$prob = (R_* + R_\mathrm{P})/a(1-e^2)$	geometric probability of transit
$b = a\cos i$	impact parameter
$s(t) = a(\sin^2\omega t + \cos^2 i\cos^2\omega t)^{1/2}$	distance between centres of star/planet discs
$\xi(t) = (a/R_*)(\sin^2\omega t + \cos^2 i\cos^2\omega t)^{1/2}$	fractional distance between centres of star/planet discs
$T_\mathrm{dur} = (P/\pi)\sin^{-1}\left[((R_* + R_\mathrm{P})^2 - a^2\cos^2 i)^{1/2}/a\right]$	duration of transit
$A_\mathrm{e} = R_*^2\left(p^2\alpha_1 + \alpha_2 - \left(4\xi^2 - (1+\xi^2-p^2)^2\right)^{1/2}/2\right)$	eclipsed area if $1-p<\xi\leq 1+p$
$p = R_\mathrm{P}/R_*$	fractional radius
$\cos\alpha_1 = (p^2+\xi^2-1)/2\xi p$	angle in eclipsed area formula
$\cos\alpha_2 = (1+\xi^2-p^2)/2\xi$	angle in eclipsed area formula
$T_\mathrm{eq} = \dfrac{1}{2}\left[\dfrac{(1-A)L_*}{\sigma\pi a^2}\right]^{1/4}$	equilibrium temperature
$T_\mathrm{day}^4 = (1-P)(1-A)(R_*^2/2a^2)T_\mathrm{eff}^4$	equilibrium day-side temperature
$T_\mathrm{night}^4 = P(1-A)(R_*^2/2a^2)T_\mathrm{eff}^4$	equilibrium night-side temperature
$\varepsilon_\lambda = p_\lambda(R_\mathrm{P}/a)^2$	amplitude of reflected light spectrum
$\Delta F_\mathrm{SE}/F \approx (T_\mathrm{day}/T_\mathrm{bright})(R_\mathrm{P}/R_*)^2$	secondary eclipse depth (Rayleigh–Jeans approx)
$P_\mathrm{lib} \sim 0.5j^{-4/3}\mu_2^{-2/3}P_2$	libration period for resonant orbit TTV

$$\delta_2 \sim \frac{P_2}{4.5j}\frac{M_1}{M_1+M_2}$$ transit timing variation (TTV) for resonant orbits

$$\frac{\mathrm{d}N_\mathrm{P,trans}}{\mathrm{d}a\,\mathrm{d}M_\mathrm{P}} \approx \frac{\theta^2}{24\pi^{9/4}} \times \left(\frac{AQ\,\Delta\lambda\,\xi t}{l_\mathrm{sky}}\right)^{3/4} \times \left(\frac{\eta\overline{\lambda}}{h\,c\,\sigma_\mathrm{FWHM}\,LSN}\right)^{3/2} \times \frac{R_\mathrm{P}^3}{a^{7/4}}\alpha_\mathrm{P}(a, M_\mathrm{P}) \times \frac{nL_*^{3/2}\exp(-3K\,d_\mathrm{max}/2)}{R_*^{5/4}}$$ transit detection probability

$$A_\mathrm{S} = V_\mathrm{S}\sin i_\mathrm{S}\left(\frac{R_\mathrm{P}^2}{R_*^2 - R_\mathrm{P}^2}\right)$$ amplitude of Rossiter–McLaughlin effect

$$g = \frac{GM_\mathrm{P}}{R_\mathrm{P}^2} = \frac{2\pi}{P}\frac{(1-e^2)^{1/2}A_\mathrm{RV}}{\left(\dfrac{R_\mathrm{P}}{a}\right)^2\sin i}$$ surface gravity

$$\frac{f_{\mathrm{day},\lambda}}{f_{*,\lambda}} = p_\lambda\left(\frac{R_\mathrm{P}}{a}\right)^2 + \frac{B_\lambda(T_\mathrm{day})}{B_\lambda(T_\mathrm{bright})}\left(\frac{R_\mathrm{P}}{R_*}\right)^2$$ planet–star flux ratio

$$T_\mathrm{day} = \frac{hc}{\lambda k}\left[\log_\mathrm{e}\left[\left(\exp\left(\frac{hc}{\lambda kT_\mathrm{bright}}\right) - 1\right)\frac{F}{\Delta F_\mathrm{SE}}\left(\frac{R_\mathrm{P}}{R_*}\right)^2 + 1\right]\right]^{-1}$$ day-side temperature

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